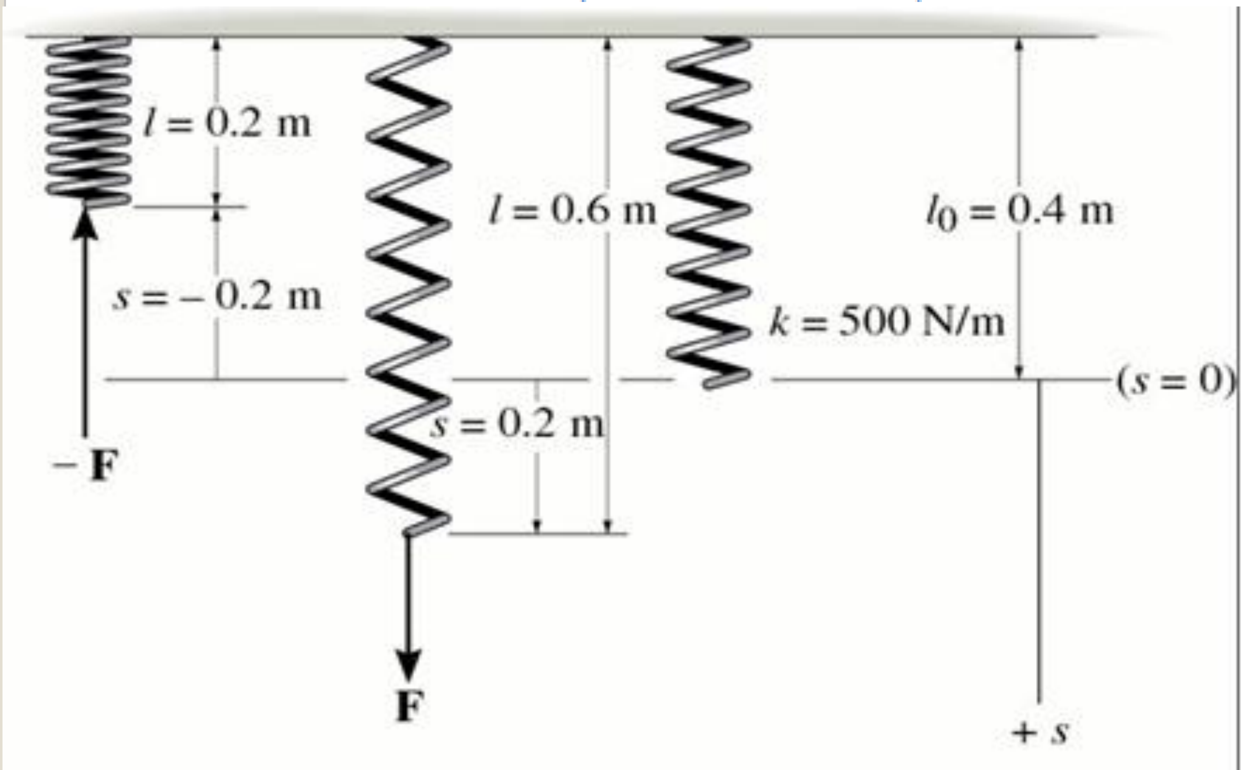
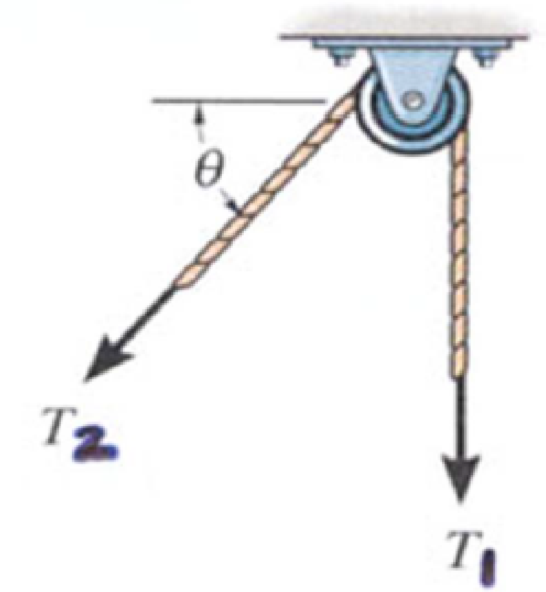
3 Equilibrium of a Particle

محمود عبد المولى

3.2 The Free-Body Diagram

SPRINGS, CABLES, AND PULLEYS





Cable is in tension

Spring Force = spring constant * deformation, or

$F = k * S$

With a frictionless pulley, $T_1 = T_2$.

- **Coplanar Systems**
- **Procedure for Analysis**

\- Free-Body Diagram

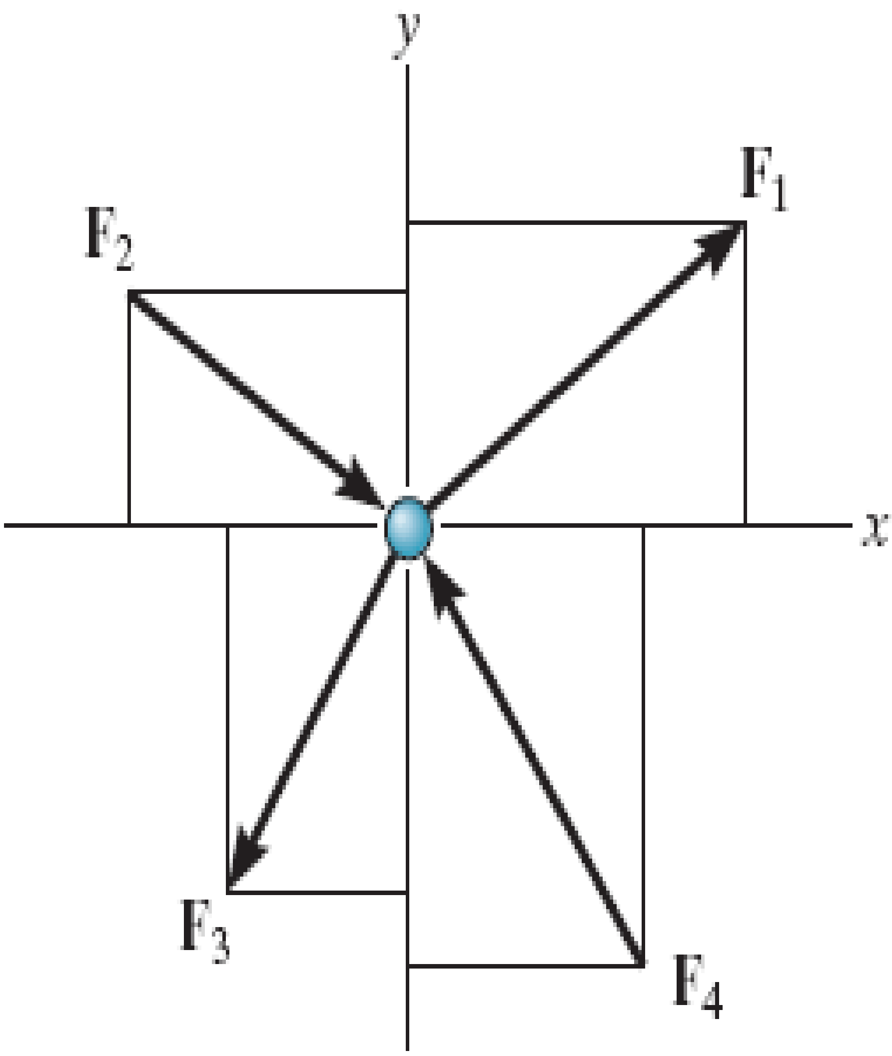
\- Establish the x, y axes

\- Label all the unknown and known forces

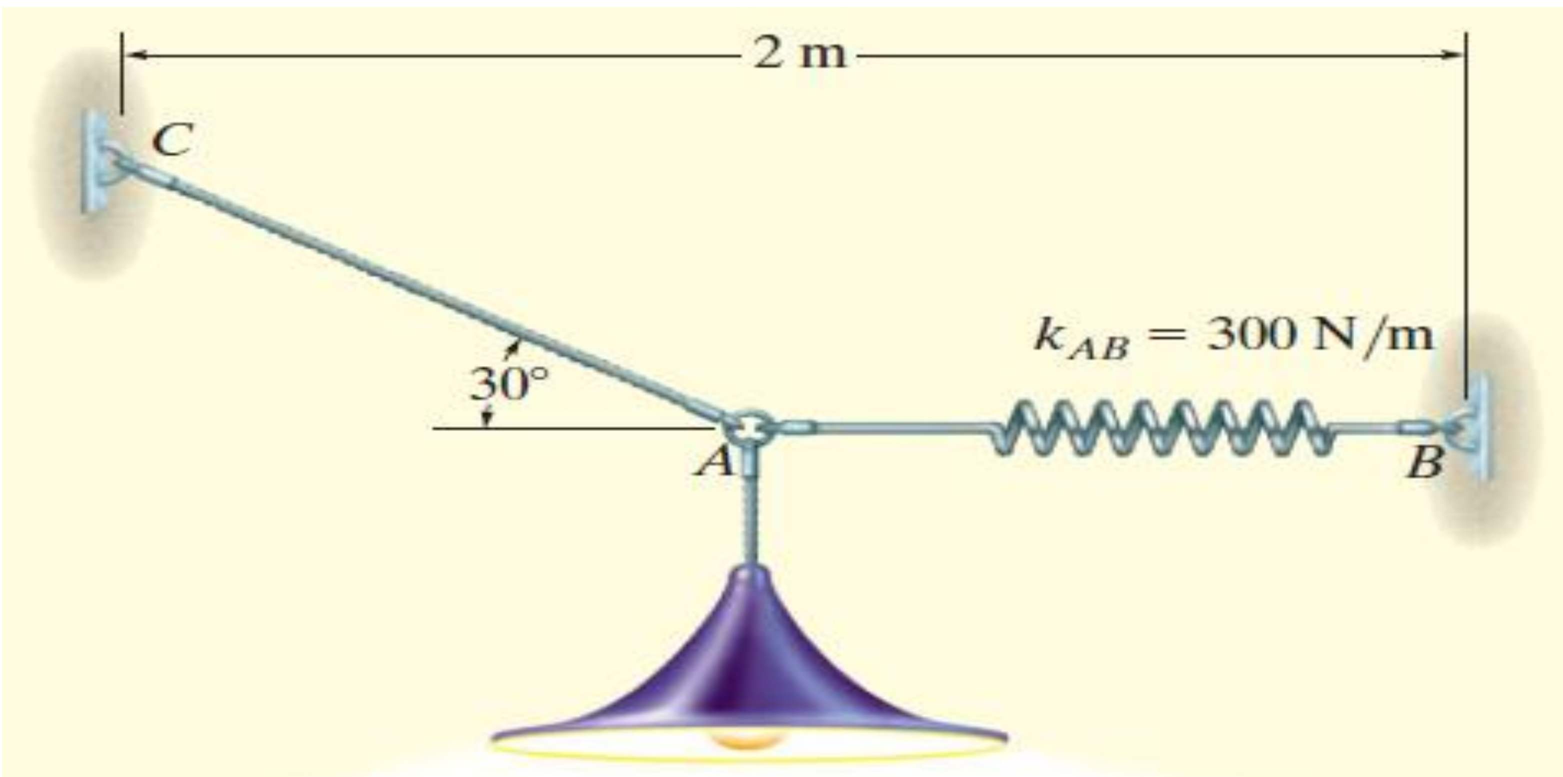
\- Resolve all forces in x, y directions

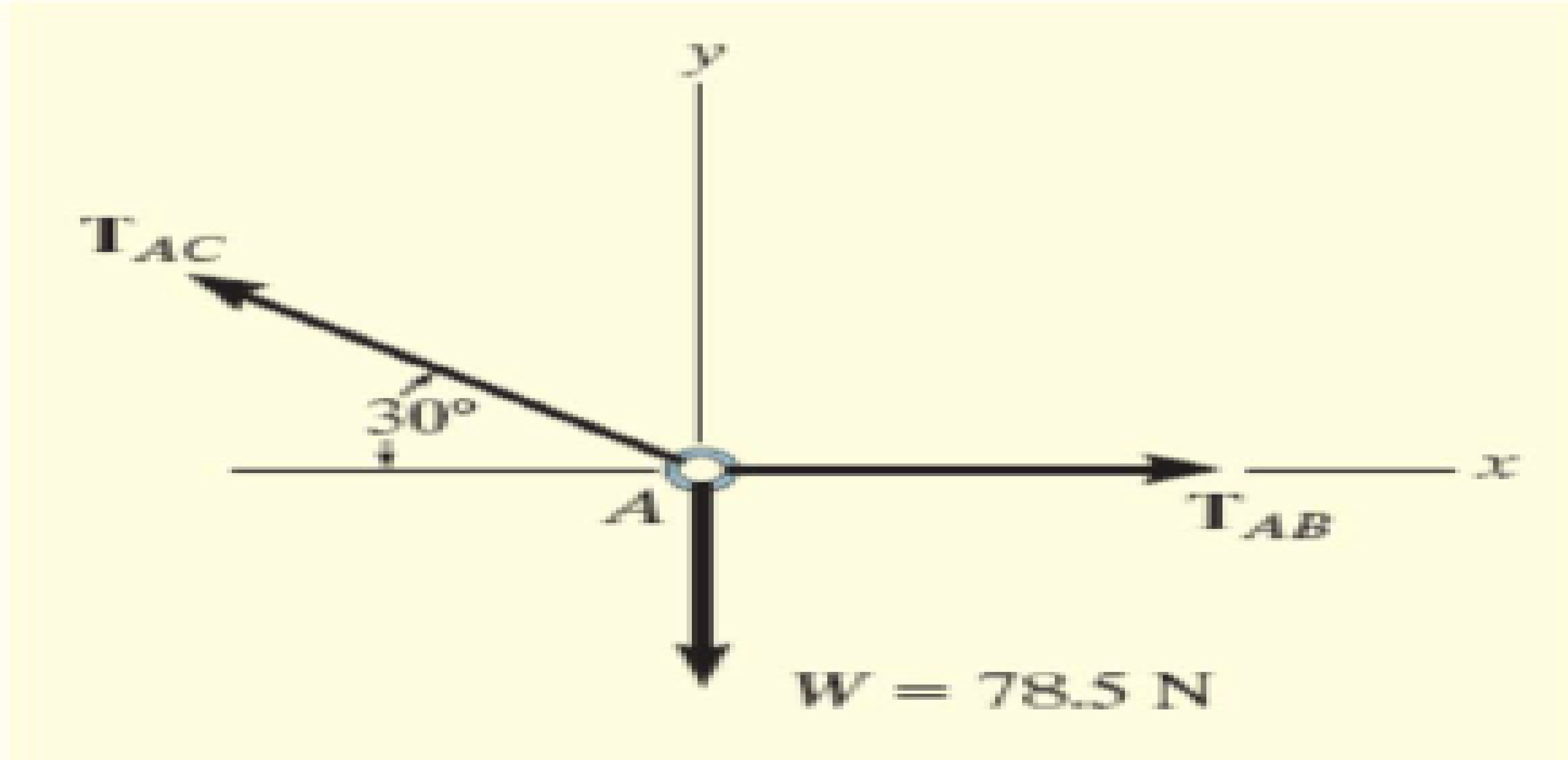
\- Apply the equations of equilibrium

$\Sigma F_x = 0$ **$\Sigma F_y = 0$**



Determine the required length of the cord AC so that the 8kg lamp is suspended. The undeformed length of the spring AB is $l'_{AB} = 0.4$m, and the spring has a stiffness of $k_{AB} = 300$N/m.





$+\rightarrow \quad \sum F_x = 0; \quad T_{AB} - T_{AC}\cos 30^{\circ} = 0$

$+\uparrow \quad \sum F_y = 0; \quad T_{AB}\sin 30^{\circ} - 78.5N = 0$

Solving,

$T_{AC} = 157.0\text{kN}$

$T_{AB} = 136.0\text{kN}$

FBD at Point A

Three forces acting, force by cable AC, force in spring AB and weight of the lamp.

If force on cable AB is known, stretch of the spring is found by $F = ks$.

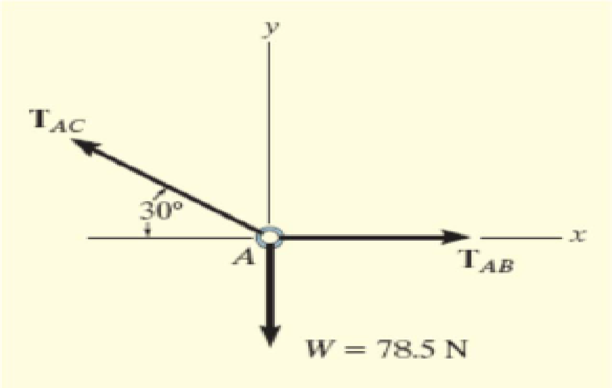
$+\rightarrow \quad \sum F_x = 0; \quad T_{AB} - T_{AC}\cos 30^\circ = 0$

$+\uparrow \quad \sum F_y = 0; \quad T_{AB}\sin 30^\circ - 78.5N = 0$

Solving,

$T_{AC} = 157.0\text{kN}$

$T_{AB} = 136.0\text{kN}$



$T_{AB} = k_{AB}s_{AB}; \; 136.0N = 300\text{N/m}(s_{AB})$

$s_{AB} = 0.453\text{m}$

For stretched length,

$l_{AB} = l'_{AB} + s_{AB}$

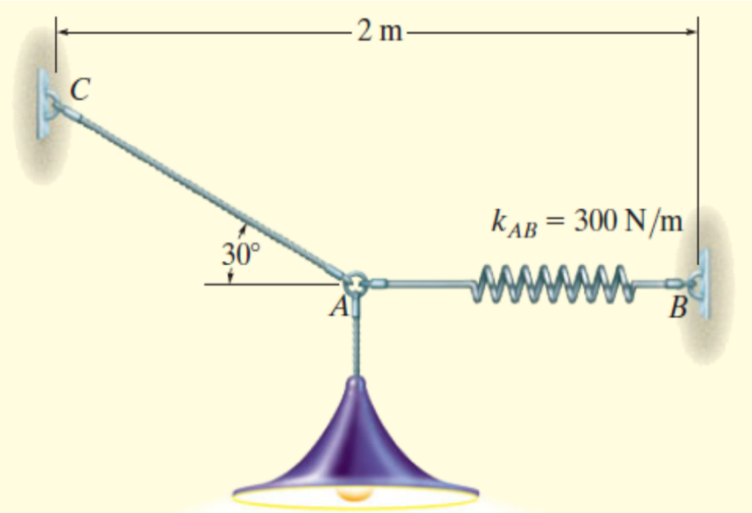
$l_{AB} = 0.4\text{m} + 0.453\text{m}$

$= 0.853\text{m}$

For horizontal distance BC,

$2\text{m} = l_{AC}\cos 30^\circ + 0.853\text{m}$

$l_{AC} = 1.32\text{m}$

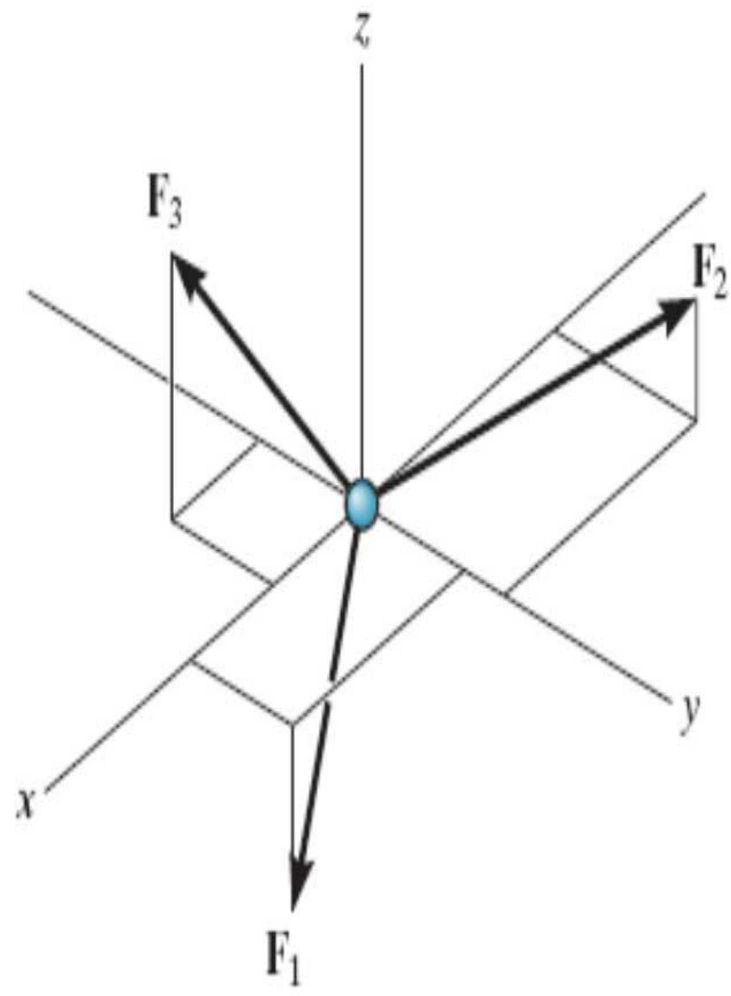


- Three-Dimensional Force Syste

• Procedure for Analysis

Free-body Diagram

- Establish the z, y, z axes
- Label all known and unknowr
- Put all forces in a vector form



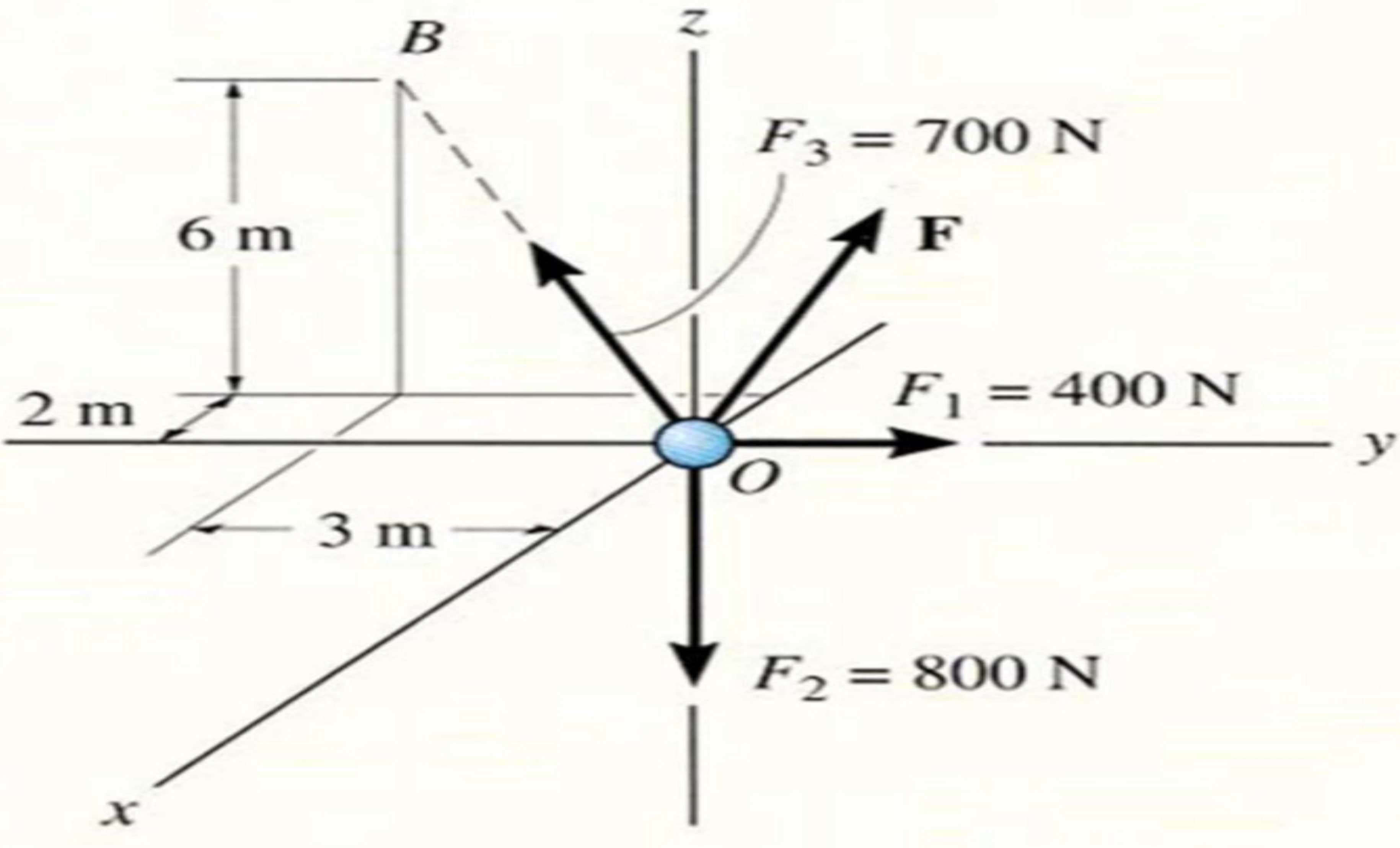
Equations of Equilibrium

- Apply

$\Sigma F_x = 0$, $\Sigma F_y = 0$ **and** $\Sigma F_z = 0$

- Substitute vectors into $\Sigma \mathbf{F} = 0$ and set **i**, **j**, **k** components = 0

Given:F1, F2 and F3. Find: The force F required to keep particle O in equilibrium



- **Given:F1, F2 and F3.**

Find: The force F required to keep particle O in equilibrium.

SOLUTION.

Put all forces in a vector form

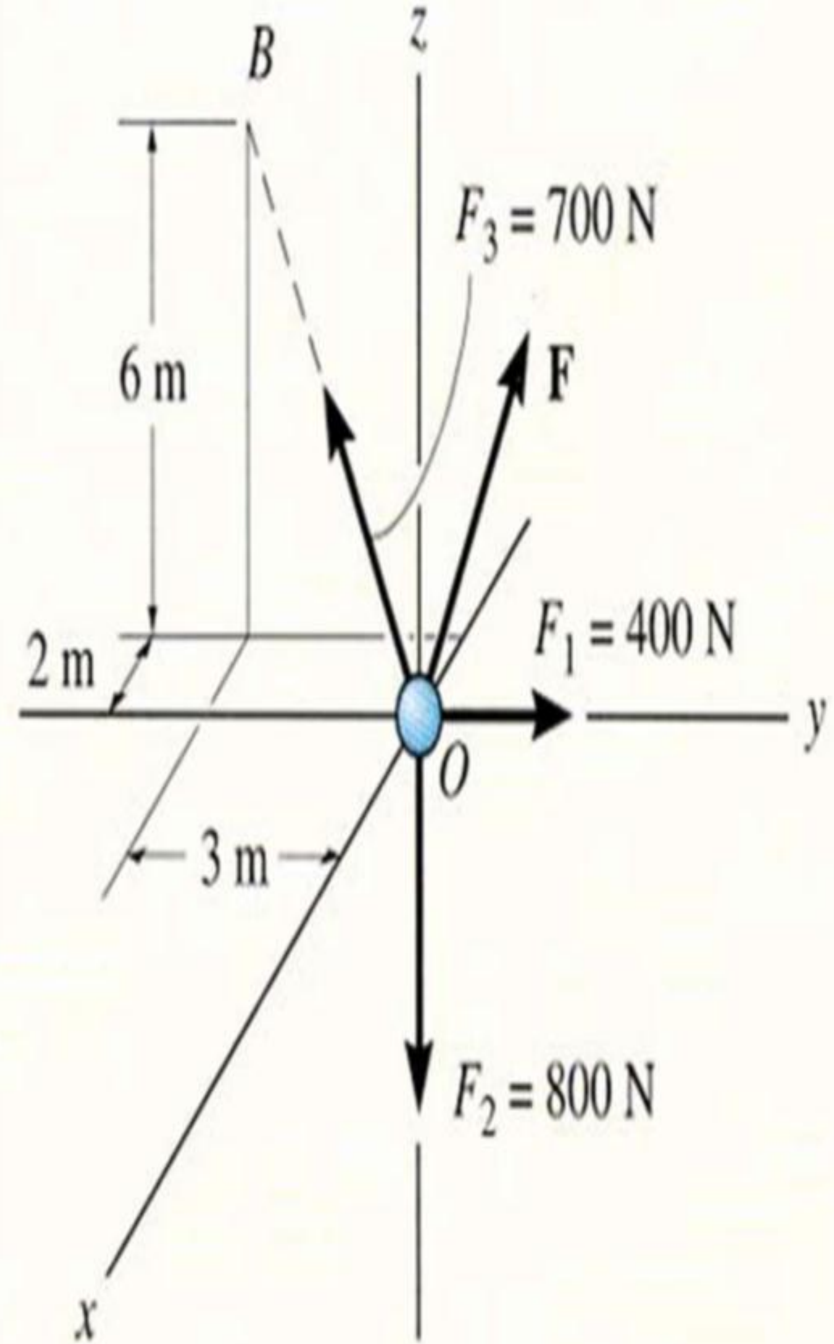
$\overrightarrow{\mathbf{F1}}$ = {400 **j**}N

$\overrightarrow{\mathbf{F2}}$ = {-800 **k**}N

$$\overrightarrow{\mathbf{F}_3} = \mathrm{F}_3 * \frac{\overrightarrow{\mathbf{0B}}}{|\overrightarrow{\mathbf{0B}}|} = 700 * \frac{(-2,-3,6)}{\sqrt{2^2+3^2+6^2}}$$

= {-200 **i** – 300 **j** + 600 **k**} N

$\overrightarrow{\mathbf{F}}$ = {$\mathbf{F_x}$ **i** + $\mathbf{F_y}$ **j** + $\mathbf{F_z}$ **k**} N



- Given:F1, F2 and F3.

Find: The force F required to keep particle O in equilibrium.

$\overrightarrow{\mathbf{F1}}$ = {400 **j**}N

$\overrightarrow{\mathbf{F2}}$ = {-800 **k**}N

$$\overrightarrow{\mathbf{F3}} = F\mathbf{3} * \frac{\overrightarrow{\mathbf{0B}}}{\left|\overrightarrow{\mathbf{0B}}\right|} = 700 * \frac{(-2,-3,6)}{\sqrt{2^2+3^2+6^2}}$$

= {-200 **i** – 300 **j** + 600 **k**} N

$\overrightarrow{\mathbf{F}}$ = {**Fx i** + **Fy j** + **Fz k**} N

For equillibruim at O

ΣFx = -200 + **Fx** = 0 ; **Fx** = 200 N

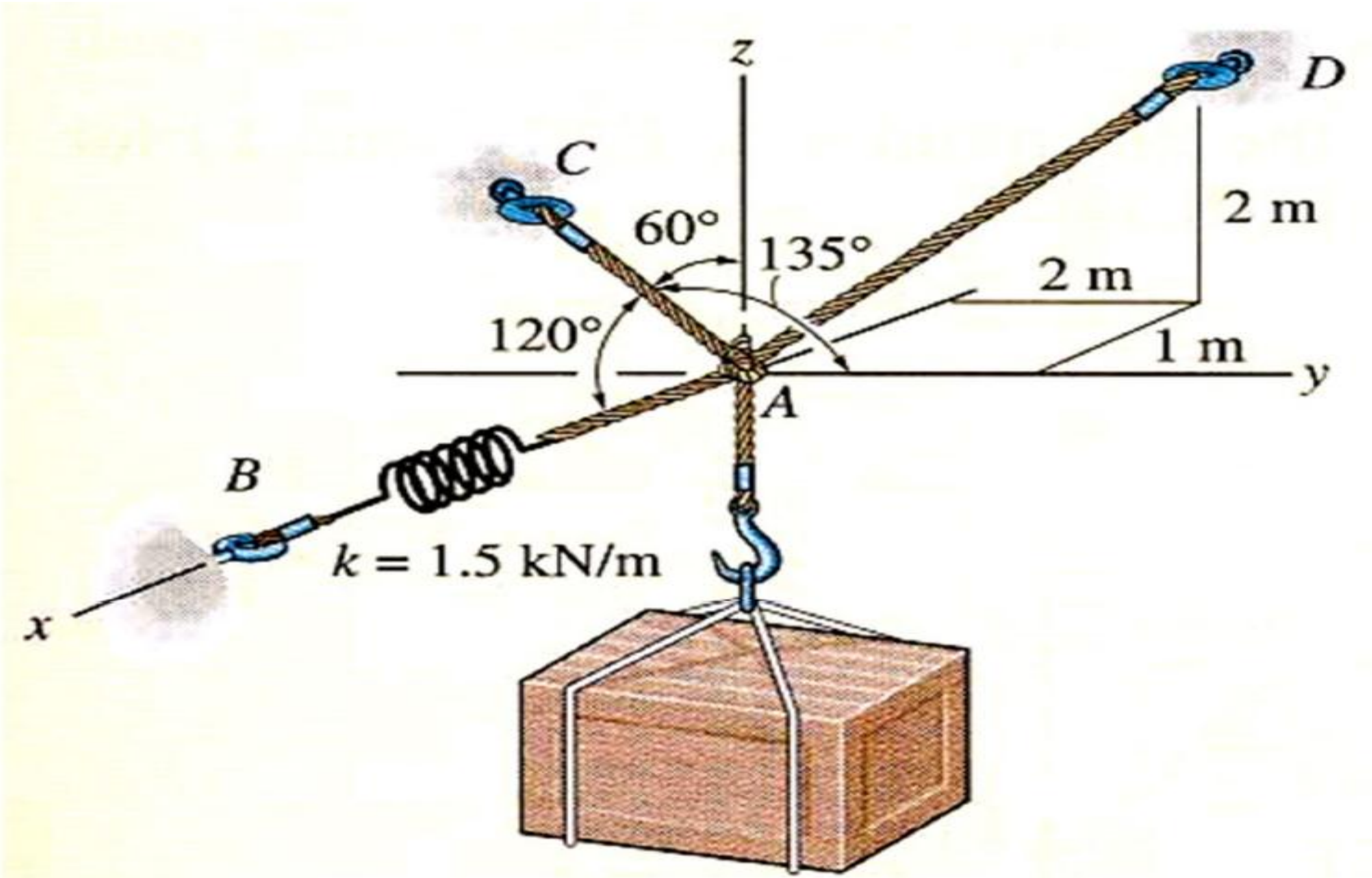
ΣFy = 400 – 300 + **Fy** = 0 ; **Fy** = -100 N

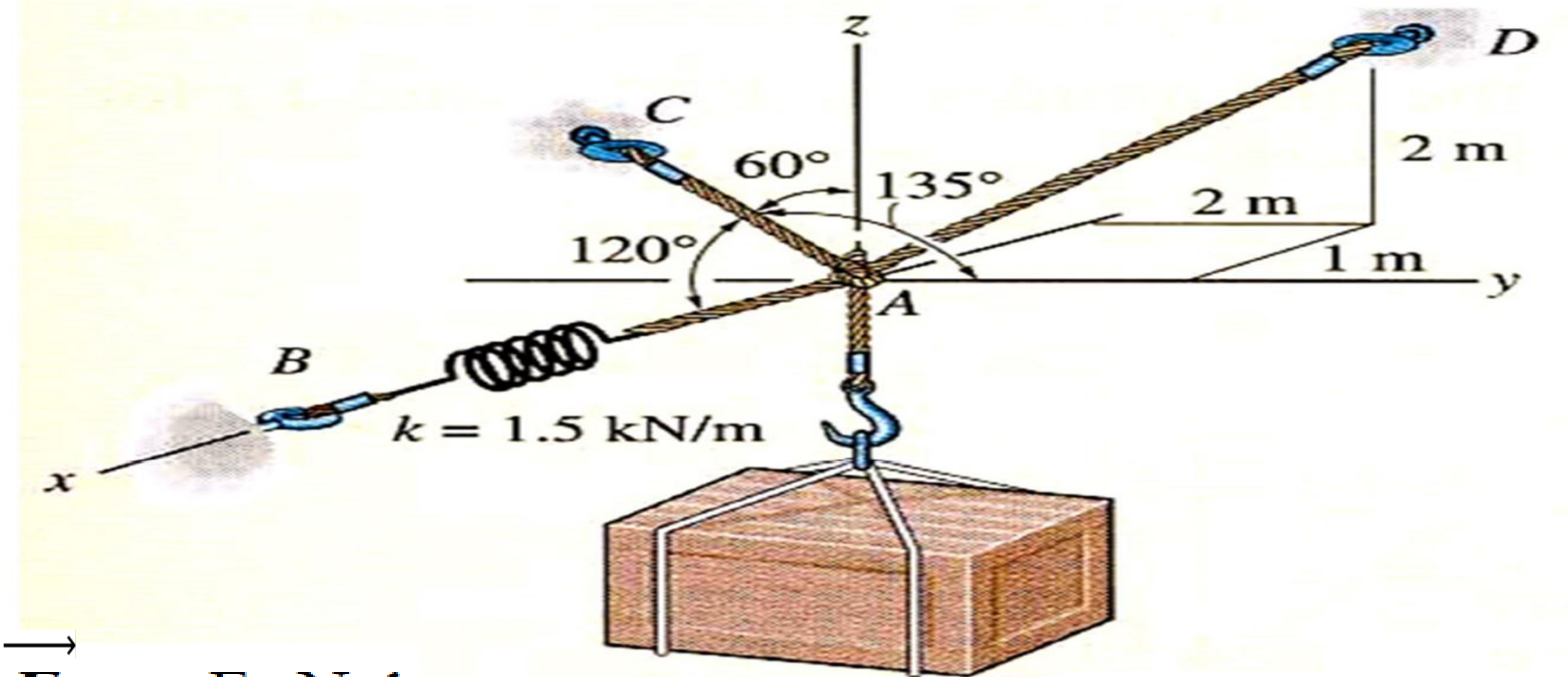
ΣFz = -800 + 600 + **Fz** = 0 ; **Fz** = 200 N

Thus, $\overrightarrow{\mathbf{F}}$ = {200 **i** – 100 **j** + 200 **k**} N

- **Given: A 100 Kg crate, as shown, is supported by three cords. One cord has a spring in it.**

Find: Tension in cords AC and AD and the stretch of the spring.





$\overrightarrow{F_B} = F_B \text{ N } \boldsymbol{i}$

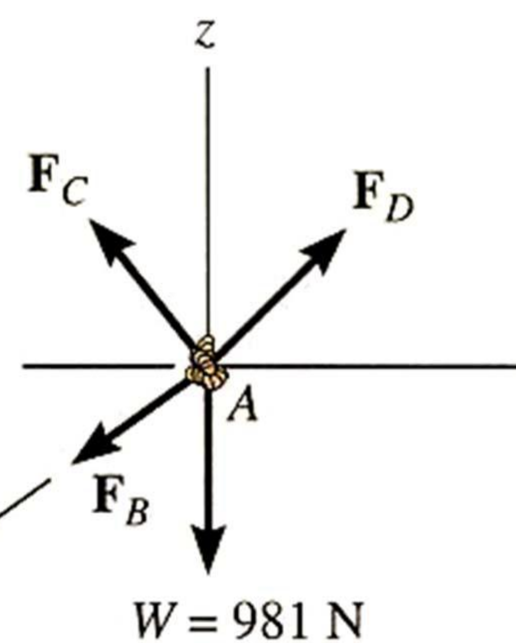
$\overrightarrow{F_C} = F_C \text{ N } (\cos 120° \, \boldsymbol{i} + \cos 135° \, \boldsymbol{j} + \cos 60° \, \boldsymbol{k})$

$= \{- 0.5 \, F_C \, \boldsymbol{i} - 0.707 \, F_C \, \boldsymbol{j} + 0.5 \, F_C \, \boldsymbol{k}\} \text{ N}$

$$\overrightarrow{F_D} = F_D * \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = F_D * \frac{(-1,2,2)}{\sqrt{1^2+2^2+2^2}}$$

$= \{- 0.3333 \, F_D \, \boldsymbol{i} + 0.667 \, F_D \, \boldsymbol{j} + 0.667 \, F_D \, \boldsymbol{k}\} \text{N}$

$\overrightarrow{W} = (- mg) \, \boldsymbol{k} = (-100 \text{ kg} * 9.81 \text{ m/sec}^2) \, \boldsymbol{k} = \{- 981 \, \boldsymbol{k}\} \text{ N}$



$$\overrightarrow{F_B} = F_B \text{ N } \boldsymbol{i}$$

$$\overrightarrow{F_C} = F_C \text{ N } (\cos 120° \, \boldsymbol{i} + \cos 135° \, \boldsymbol{j} + \cos 60° \, \boldsymbol{k})$$

$$= \{-0.5 \, F_C \, \boldsymbol{i} - 0.707 \, F_C \, \boldsymbol{j} + 0.5 \, F_C \, \boldsymbol{k}\} \text{ N}$$

$$\overrightarrow{F_D} = F_D * \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = F_D * \frac{(-1,2,2)}{\sqrt{1^2+2^2+2^2}}$$

$$= \{-0.3333 \, F_D \, \boldsymbol{i} + 0.667 \, F_D \, \boldsymbol{j} + 0.667 \, F_D \, \boldsymbol{k}\} \text{N}$$

$$\overrightarrow{W} = (-\text{mg}) \, \boldsymbol{k} = (-100 \text{ kg} * 9.81 \text{ m/sec}^2) \, \boldsymbol{k} = \{-981 \, \boldsymbol{k}\} \text{ N}$$

Now equate the respective ***i***, ***j***, ***k*** components to zero.

$$\Sigma F_x = F_B - 0.5 F_C - 0.333 F_D = 0$$

$$\Sigma F_y = -0.707 \, F_C + 0.667 \, F_D = 0$$

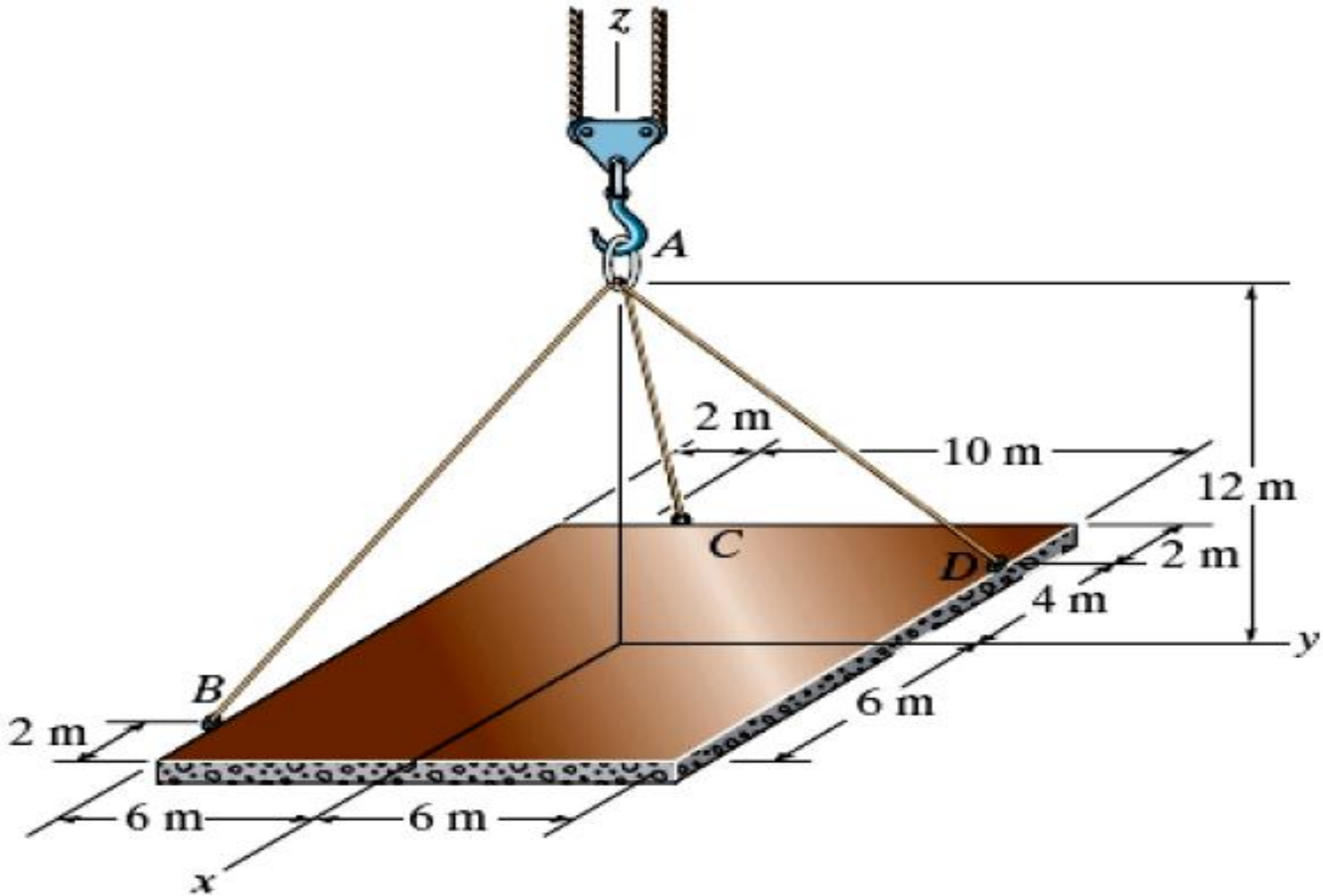
$$\Sigma F_z = 0.5 \, F_C + 0.667 \, F_D - 981 \text{ N} = 0$$

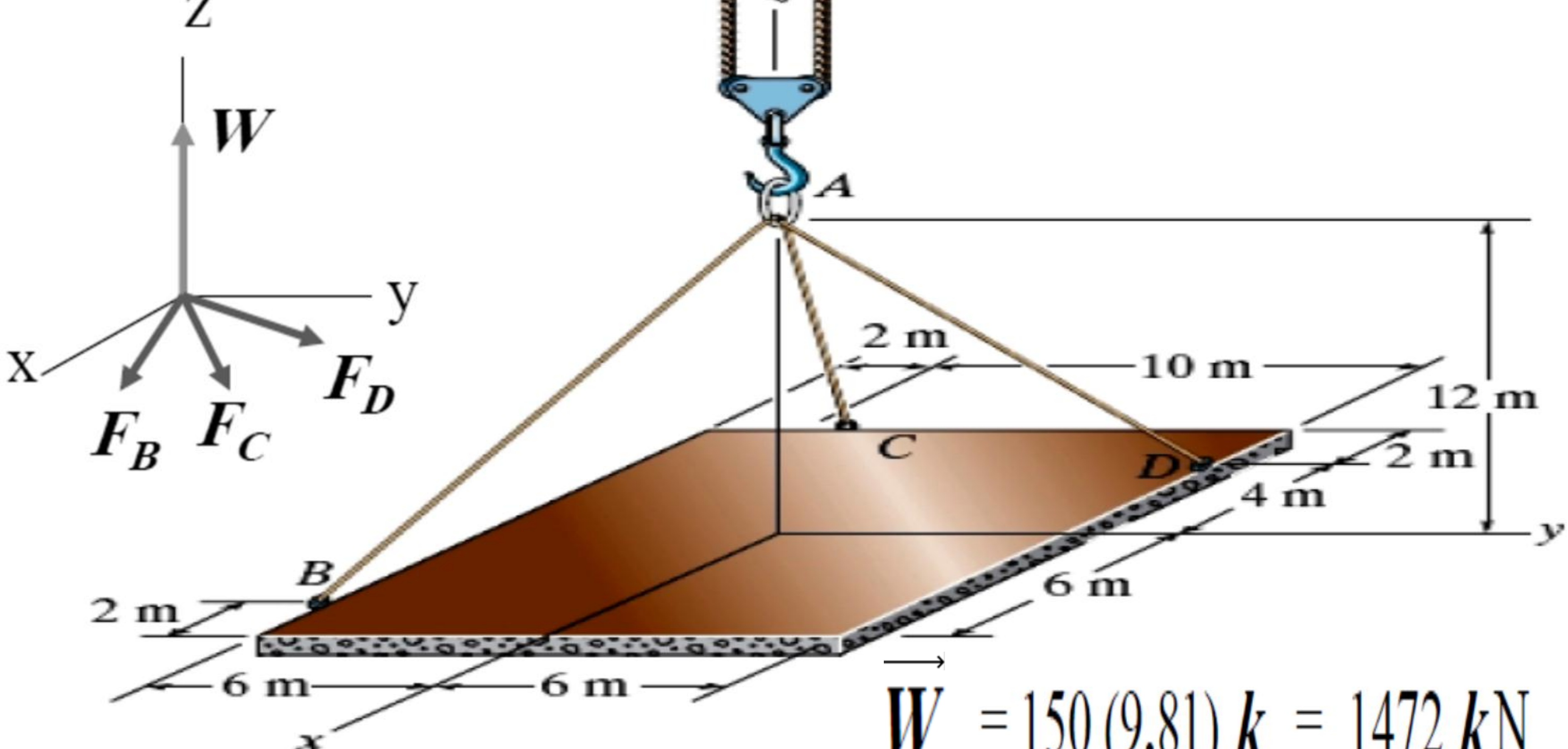
$F_C = 813$ N $\quad F_D = 862$ N $\quad F_B = 693.7$ N

The spring stretch is (from $F = k * s$)

$$s = F_B / k = 693.7 \text{ N} / 1500 \text{ N/m} = 0.462 \text{ m}$$

Given: A 150 Kg plate, as shown, is supported by three cables and is in equilibrium. Find: Tension in each of the cables.





$$\overrightarrow{W} = 150\,(9.81)\,k = 1472\,k\text{N}$$

$$\overrightarrow{F_B} = F_B * \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = F_B * \frac{(4,-6,-12)}{\sqrt{4^2+6^2+12^2}} = F_B * \frac{(4,-6,-12)}{14}$$

$$\overrightarrow{F_C} = F_C * \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = F_C * \frac{(-6,-4,-12)}{\sqrt{6^2+4^2+12^2}} = F_C * \frac{(-6,-4,-12)}{14}$$

$$\overrightarrow{F_D} = F_D * \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = F_D * \frac{(-4,6,-12)}{\sqrt{4^2+6^2+12^2}} = F_D * \frac{(-4,6,-12)}{14}$$

$$\vec{W} = 150\,(9.81)\,\boldsymbol{k} = 1472\,\boldsymbol{k}\,\text{N}$$

$$\vec{F_B} = F_B * \frac{\vec{AB}}{|\vec{AB}|} = F_B * \frac{(4,-6,-12)}{\sqrt{4^2+6^2+12^2}} = F_B * \frac{(4,-6,-12)}{14}$$

$$\vec{F_C} = F_C * \frac{\vec{AC}}{|\vec{AC}|} = F_C * \frac{(-6,-4,-12)}{\sqrt{6^2+4^2+12^2}} = F_C * \frac{(-6,-4,-12)}{14}$$

$$\vec{F_D} = F_D * \frac{\vec{AD}}{|\vec{AD}|} = F_D * \frac{(-4,6,-12)}{\sqrt{4^2+6^2+12^2}} = F_D * \frac{(-4,6,-12)}{14}$$

$$\sum F_x = (4/14)F_B - (6/14)F_C - (4/14)F_D = 0$$

$$\sum F_y = (-6/14)F_B - (4/14)F_C + (6/14)F_D = 0$$

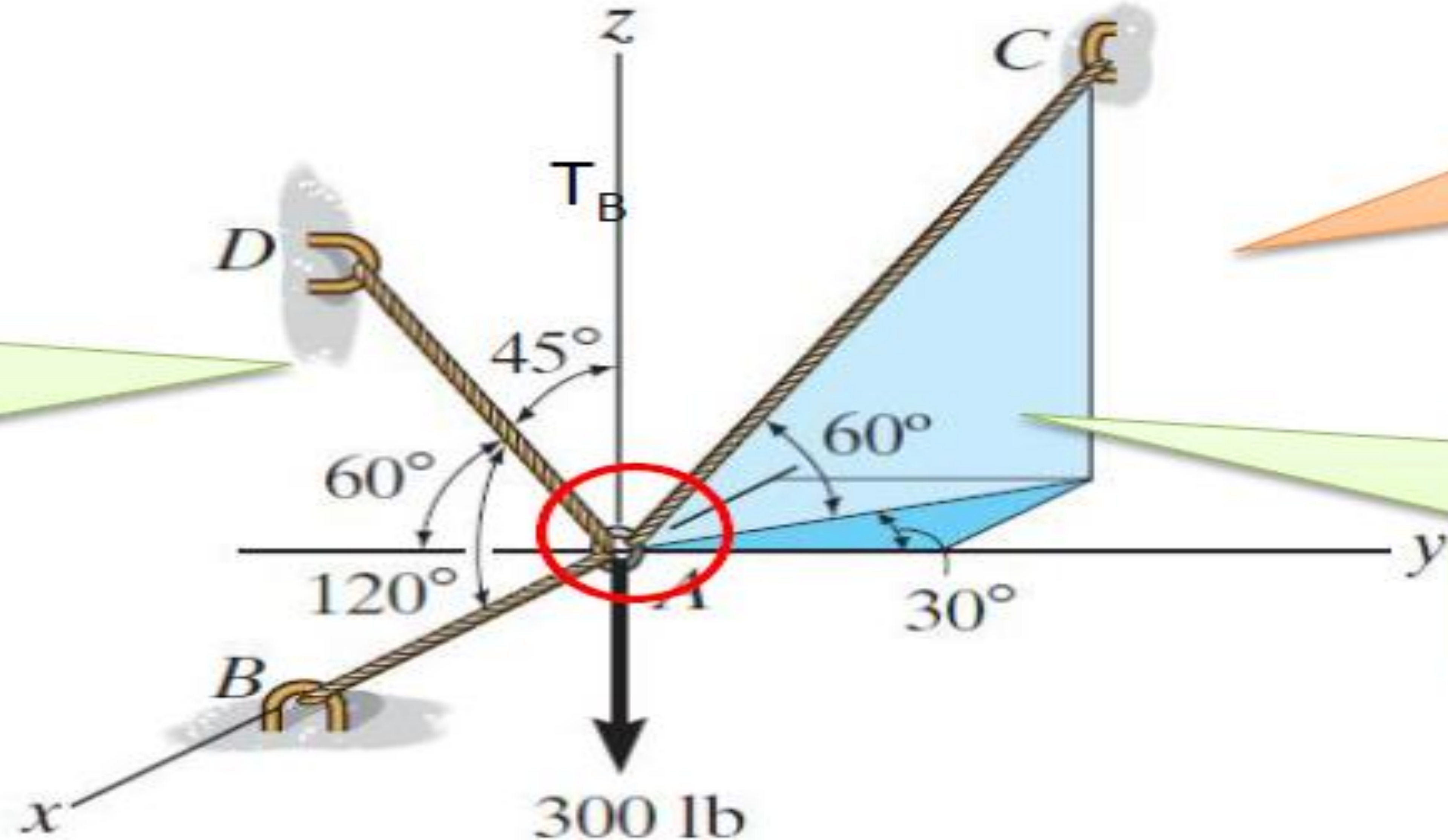
$$\sum F_z = (-12/14)F_B - (12/14)F_C - (12/14)F_D + 1472 = 0$$

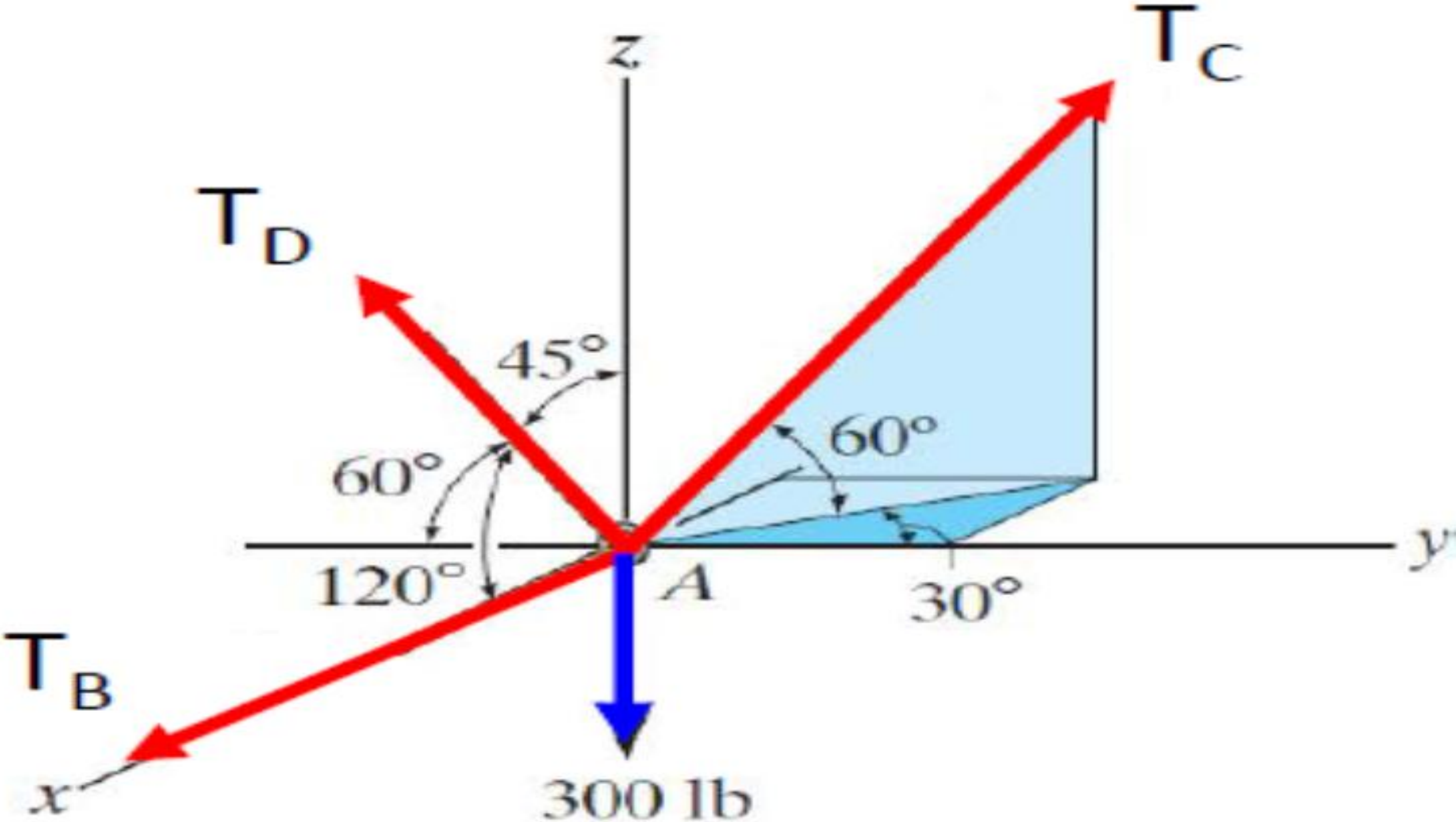
F_B =858.67 N

F_C =0 N

F_D =858.67 N

If the whole assembly is in equilibrium ,determine the tension developed in each cables





$$\overrightarrow{T_C} = -(T_C \cos 60°) \sin 30°\, i + (T_C \cos 60°) \cos 30°\, j + T_C \sin 60°\, k$$

$$\overrightarrow{W} = -300\, \boldsymbol{k} \qquad \overrightarrow{T_B} = T_B\ \boldsymbol{i}$$

$$\overrightarrow{T_D} = T_D \cos 120°\, i + T_D \cos 120°\, j + T_D \cos 45°\, k$$

$$\overrightarrow{T_B} = T_B\ \boldsymbol{i}$$

$$\overrightarrow{T_C} = -(T_C \cos 60°) \sin 30°\ \boldsymbol{i} + (T_C \cos 60°) \cos 30°\ \boldsymbol{j} + T_C \sin 60°\ \boldsymbol{k}$$

$$\overrightarrow{T_C} = T_C\,(-0.25\ \boldsymbol{i} + 0.433\ \boldsymbol{j} + 0.866\ \boldsymbol{k})$$

$$\overrightarrow{T_D} = T_D \cos 120°\ \boldsymbol{i} + T_D \cos 120°\ \boldsymbol{j} + T_D \cos 45°\ \boldsymbol{k}$$

$$\overrightarrow{T_D} = T_D\,(-0.5\ \boldsymbol{i} - 0.5\ \boldsymbol{j} + 0.7071\ \boldsymbol{k})$$

$$\overrightarrow{W} = -300\ \boldsymbol{k}$$

Equating the respective ***i***, ***j***, ***k*** components to zero,

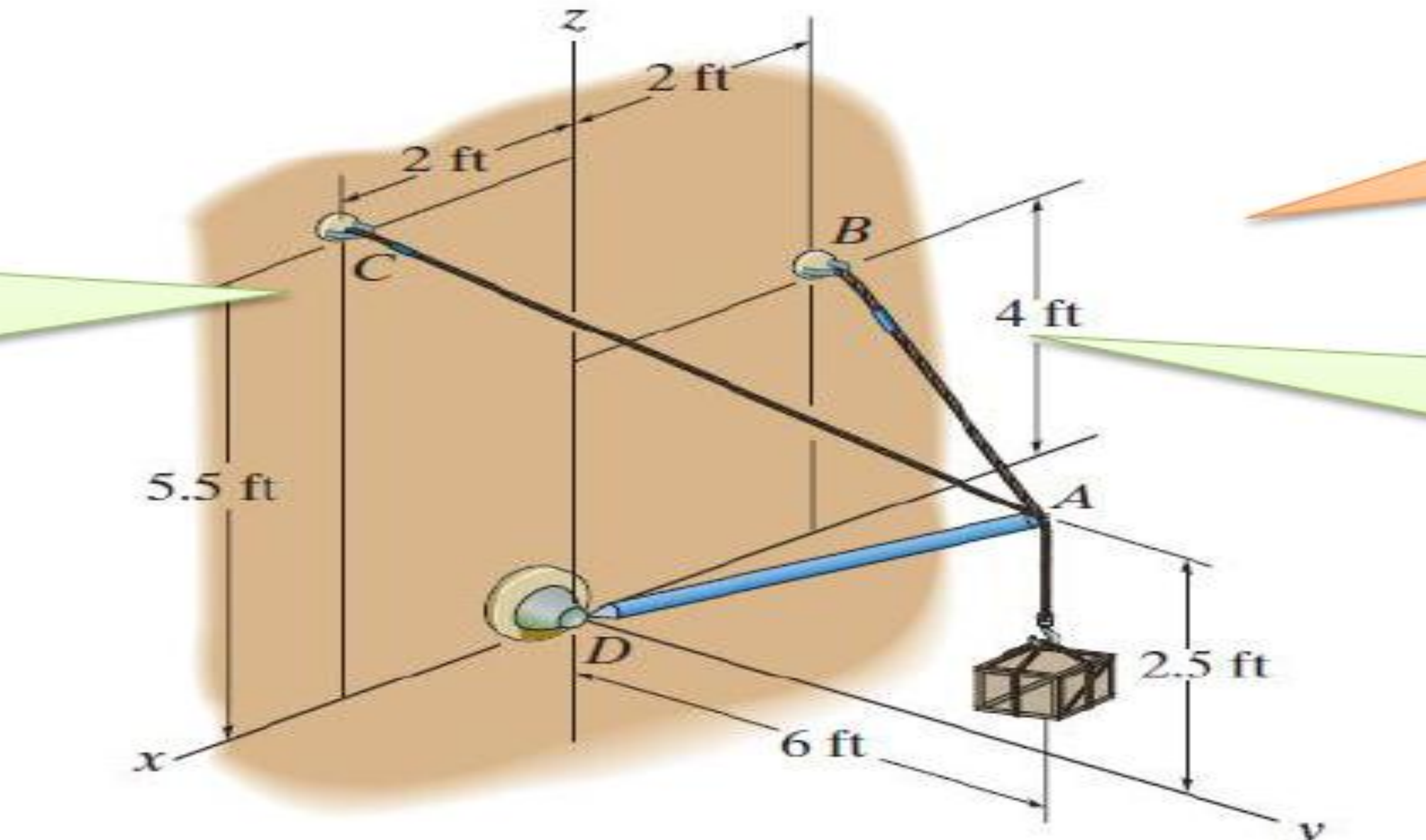
$\Sigma F_x = T_B - 0.25\ T_C - 0.5\ T_D = 0$ (1)

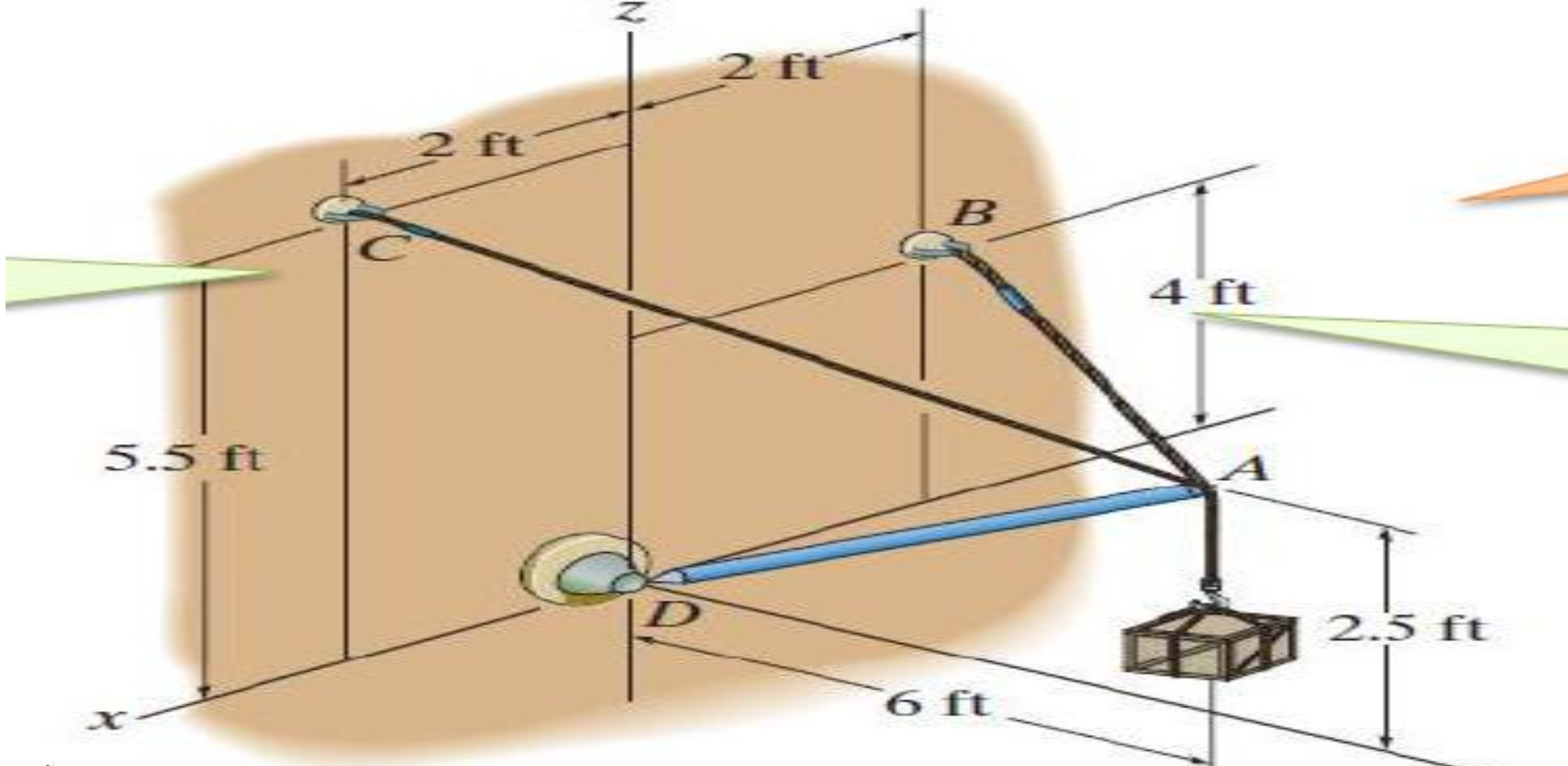
$\Sigma F_y = 0.433\ T_C - 0.5\ T_D = 0$ (2)

$\Sigma F_z = 0.866\ T_C + 0.7071\ T_D - 300 = 0$ (3)

Using (2) and (3), T_C = 203 lb, T_D = 176 lb
Substituting T_C and T_D into (1), T_B = 139 lb

- **If the whole assembly is in equilibrium ,and supported by two cables and strut AD .Given 400 lb crate, determine the magnitude of the tension developed in each cables and the force developed along strut AD.**

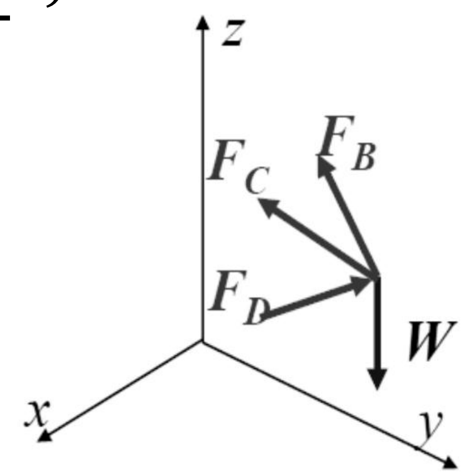




$$\overrightarrow{F_B} = F_B * \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = F_B * \frac{(-2,-6,1.5)}{\sqrt{2^2+6^2+1.5^2}} = F_B * \frac{(-2,-6,1.5)}{6.5}$$

$$\overrightarrow{F_C} = F_C * \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = F_C * \frac{(2,-6,3)}{\sqrt{2^2+6^2+3^2}} = F_C * \frac{(2,-6,3)}{7}$$

$$\overrightarrow{F_D} = F_D * \frac{\overrightarrow{DA}}{|\overrightarrow{DA}|} = F_D * \frac{(0,6,2.5)}{\sqrt{6^2+2.5^2}} = F_D * \frac{(0,6,2.5)}{6.5}$$

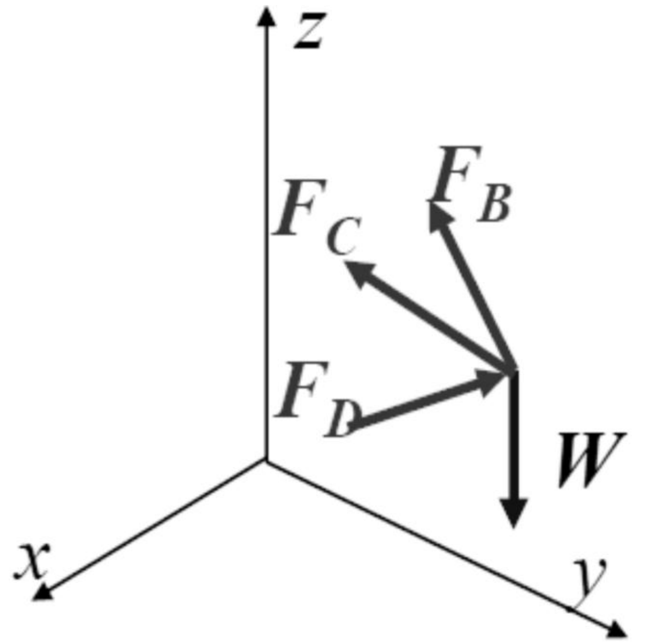


$\overrightarrow{W}$ = weight of crate = - 400 $\boldsymbol{k}$ lb

$$\overrightarrow{F_B} = F_B * \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = F_B * \frac{(-2,-6,1.5)}{\sqrt{2^2+6^2+1.5^2}} = F_B * \frac{(-2,-6,1.5)}{6.5}$$

$$\overrightarrow{F_C} = F_C * \frac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = F_C * \frac{(2,-6,3)}{\sqrt{2^2+6^2+3^2}} = F_C * \frac{(2,-6,3)}{7}$$

$$\overrightarrow{F_D} = F_D * \frac{\overrightarrow{DA}}{|\overrightarrow{DA}|} = F_D * \frac{(0,6,2.5)}{\sqrt{6^2+2.5^2}} = F_D * \frac{(0,6,2.5)}{6.5}$$



$$\sum F_X \quad \frac{-2}{6.5}F_B + \frac{2}{7}F_C = 0 \quad (1)$$

$$\sum F_Y \quad \frac{-6}{6.5}F_B - \frac{6}{7}F_C + \frac{6}{6.5}F_D = 0 \quad (2)$$

$$\sum F_Z \quad \frac{1.5}{6.5}F_B + \frac{3}{7}F_C + \frac{2.5}{6.5}F_D = 400 \quad (3)$$

F_B=274.14127 , F_C=295.30306 , F_D=548.3284